

**3.4.9**

(a)  $m_X(s) = E(e^{sX}) = \int_0^{10} e^{sx}(1/10)dx = (1/10)(e^{10s} - 1)/s$  for  $s \neq 0$ , with (of course)  $m_X(0) = 1$ .

(b) For  $s \neq 0$ ,  $m'_X(s) = (1/10)(s(10e^{10s}) - (e^{10s} - 1))/s^2$ . We then compute using L'Hôpital's Rule (twice) that  $m'_X(0) = \lim_{s \rightarrow \infty} m'_X(s) = 5 = E(X)$ .

**3.4.21** The mgf of the Poisson( $\lambda_i$ ) equals  $m_i(s) = \exp\{\lambda_i(e^s - 1)\}$ . Then the mgf of  $Y = X_1 + \cdots + X_n$  is given by (Theorem 3.4.5)

$$m_Y(s) = \prod_{i=1}^n m_i(s) = \prod_{i=1}^n \exp\{\lambda_i(e^s - 1)\} = \exp\left\{\sum_{i=1}^n \lambda_i(e^s - 1)\right\}$$

and we recognize this as the mgf of the Poisson( $\sum_{i=1}^n \lambda_i$ ). Therefore, the uniqueness theorem implies that this is the distribution of  $Y$ .

**3.6.2** Since  $X \geq 0$ ,  $P(X \geq 3) \leq E(X)/3 = (1/5)/3 = 1/15$ .

**3.6.10**

(a)  $E(W) = \int_{R^1} wf(w)dw = \int_0^1 w(3w^2)dw = 3w^4/4|_{w=0}^{w=1} = 3/4$ .

(b)  $E(W^2) = \int_0^1 w^2(3w^2)dw = 3w^5/5|_{w=0}^{w=1} = 3/5$ . Thus,  $\text{Var}(W) = 3/5 - (3/4)^2 = 3/80$ . Hence, the Chebyshev's inequality bound is  $3/5$  because  $P(|W - E(W)| \geq 1/4) \leq \text{Var}(W)/(1/4)^2 = (3/80)/(1/16) = 3/5$ .

**3.6.18**

(a) We have that  $\int_1^\infty (2/x^3) dx = -x^{-2}|_1^\infty = 1$ , so  $f_X$  is a density.

(b)  $E(X) = \int_1^\infty x(2/x^3) dx = \int_1^\infty (2/x^2) dx = -2x^{-1}|_1^\infty = 2$ .

(c) Markov's inequality says that  $P(X \geq k) \leq E(X)/k = 2/k$ , while the precise value is  $P(X \geq k) = \int_k^\infty (2/x^3) dx = -x^{-2}|_k^\infty = 1/k^2$ , and we see that the tail probability declines quadratically, while Markov's inequality only declines linearly.

(d) We have that  $E(X^2) = \int_1^\infty x^2(2/x^3) dx = \int_1^\infty (2/x) dx = -2 \ln x|_1^\infty = \infty$ . Therefore,  $\text{Var}(X) = \infty$  and Chebyshev's inequality does not provide a useful bound in this case.